# **Characteristics Analysis of the Square Laminated Core under DC-biased Magnetization by the Fixed-point Harmonic-Balanced FEM**

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**Abstract —The ferromagnetic core of power transformer is susceptible to DC bias and has special nonlinear and hysteretic characteristics under DC-biased magnetization. The DC bias test is carried out to obtain the DC-biasing magnetizing curve of a square laminated core (SLC). The harmonic-balanced finite element method (HBFEM) is combined with the fixedpoint technique to calculate the magnetic field in the laminated core under DC bias conditions by means of the measured DC-biasing magnetizing curve. A locally convergent method in harmonic domain is presented to determine the fixed-point reluctivity, which can speed up the convergence of harmonic solution.** 

## I. INTRODUCTION

DC flux and AC flux coexists in the ferromagnetic core when the power transformer operates under DC-biased magnetization which leads to the distortion and asymmetry of hysteresis loops. The loss curve of iron core under DCbiased excitation is also different from that under sinusoidal excitation [1]-[2]. Fig. 1 shows two different hysteresis loops under DC-biased and sinusoidal excitations.  $B_{\text{acm}}$  is the magnitude of AC component of flux density and  $B_{dc}$  is the DC component of flux density. The hysteresis loop is distorted seriously and the ferromagnetic core is saturated significantly by the DC flux density  $B_{\text{dc}}$ . As a result, the magnetizing curve under DC-biased excitation is different from that under sinusoidal excitation.

The fixed-point technique has been used in finite element analysis of nonlinear magnetic field under sinusoidal excitation [3]-[4]. Two different methods to determine the fixed-point reluctivity are presented and the convergent performance has been discussed in detail [5]. In this paper, the fixed-point technique is combined with the HBFEM to calculate the DC-biased magnetic field by means of the measured magnetizing curves of the SLC under DC-biased magnetization, neglecting the hysteretic effect and anisotropic property.



Fig. 1. Hysteresis loops under DC-biased and sinusoidal excitations



Fig. 2. The square laminated core and coils

## II. EXPERIMENT TO OBTAIN DC-BIASING MAGNETIZING **CURVE**

The magnetizing curve can be treated as the locus of vertices of hysteresis loops. Therefore the hysteresis loops are required to obtain the magnetizing curves under DCbiased magnetization. The DC-biasing experiment is carried out on the SLC shown in Fig. 2. The laminated core is entwined by two coils of both 312 turns, one is used as the exciting coil fed by alternating voltage and used for measurement of no-load exciting current *I*, and the other one is used as the search coil for measurement of the noload induced voltage  $U_{ac}$ . The DC bias current  $I_{dc}$  is applied to the exciting coil by a direct current source. To obtain the DC-biasing hysteresis loops, the measured data are manipulated by,

$$
\varphi_{ac} = (1/N_{coil})U_{ac}dt \tag{1}
$$

$$
B = \left(\varphi_{ac} + \varphi_{dc}\right) / S \tag{2}
$$

$$
H = I \cdot N / L \tag{3}
$$

where  $\varphi_{ac}$  and  $\varphi_{dc}$  are the AC and DC component of the total flux respectively in the magnetic core, *S* is the crosssectional area of the core, and *L* is the mean length of the magnetic circuit. The DC flux  $\varphi_{dc}$  cannot be measured experimentally and therefore can be predicted by the following iterative process [6]. First, *φ*ac and exciting current *I* can be measured under a given DC-biasing working condition. Then based on the initially assumed *φ*<sub>dc</sub>, the exciting current  $I_c$  can be calculated by the normal magnetizing curve. Finally  $\varphi_{dc}$  is adjusted on the basis of comparison between *I* and *I*c until the measured and calculated exciting current match.

With the calculated  $\varphi_{\text{dc}}$ , hysteresis loops under DCbiased magnetization shown in Fig.3 can be obtained by solving (1)-(3). The DC component of magnetic intensity  $H_{dc}$  can also be calculated in (3) with the given direct current  $I_{dc}$ . Table I presents the measured data, which demonstrate the asymmetry of the magnetizing curve under DC-biased magnetization.



TABLE I DATA OF MAGNETIZING CURVE UNDER DC-BIASED<br>MAGNETIZATION (H = 107A/m)

MAUNETIZATION $(H_{dc} = 10/H/\text{H})$						
$B_{\rm m}$	$-1.543$	$-1.244$	$-0.673$	$-0.276$	$-0.079$	0.114
$H_{\rm b}$	$-104.77$	32.892	$-13.986$	10.143	$-8.328$	$-7.238$
$B_{\rm m}$	0.314	0.876	1.330	1.647	1.783	1.908
$H_{\rm b}$	$-6.323$	$-0.747$	47.177	325.51	1319.6	2665.8

### III. FIXED-POINT HARMONIC-BALANCED METHOD

A new relationship between magnetic intensity and flux density is represented by introducing the fixed-point reluctivity  $v_{FP}$  [7],

$$
H(B) = v_{FP}B - M(B)
$$
 (4)

where  $v_{FP}$  is a constant value. *M* is a magnetization-like quantity which varies nonlinearly with the flux density *B*. Therefore the magnetic intensity  $H$  is divided into two parts, the linear part related to  $v_{FP}$  and nonlinear part related to M.

 The vector potential equation is used to describe the two-dimensional nonlinear magnetic field by substituting (4) into the Maxwell's equations,

$$
\nabla \times \nu_{\rm FP} (\nabla \times A) + \sigma \frac{\partial A}{\partial t} = J - \nabla \times M \tag{5}
$$

where *A* is the magnetic vector potential and  $\sigma$  is the electric conductivity.

The fixed-point reluctivity determines the convergence of solutions. If  $v_{FP}$  is set be constant, the fixed-point iteration can be very slow and even not stable especially when  $v_{FP}$  is not selected properly. Actually  $v_{FP}$  can be updated in each iterative step and a fast convergence can be achieved based on the locally convergent method. The local fixed-point reluctivity [5] can be determined by,

$$
V_{FP} = \left(\partial H_x / \partial B_x + \partial H_y / \partial B_y\right) / 2\tag{6}
$$

under DC-biased excitation can be approximated by the Fourier-series with a finite number of harmonics [8],

$$
W(t) = W_0 + \sum_{n=1}^{\infty} \left( W_{ns} \sin n\omega t + W_{nc} \cos n\omega t \right)
$$
 (7)

*IEEE Trans. on Magn.*, vol.43, no.11, pp. 3969-3975, 2007.<br>IFEE Trans. on Magn., vol.43, no.11, pp. 3969-3975, 2007.<br>ISI E. Dlala and A. Arkkio, "Analysis of the convergence of the fixedpotential *A*, flux density *B*, magnetic intensity *H*, and *M*.

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$$
v_{FP}(t) = \frac{dH(B)}{dB} = v_{d0} + \sum_{n=1}^{\infty} \left\{ v_{dns} \sin(n\omega t) + v_{dnc} \cos(n\omega t) \right\} (8)
$$

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The fixed-point harmonic-balanced equation can be obtained by substituting (7) and (8) into the weak form of (5) based upon Galerkin's method,

$$
SA + TA - GJ = P \tag{9}
$$

where *S* and *T* are related to  $v_{FP}$  in harmonic domain and eddy current respectively, and *P* is derived from magnetization-like quantity *M*.

The electromagnetic coupling can be considered by,

$$
U_{\text{in}} = CA + ZJ \tag{10}
$$

Fig. 3. Hysteresis loops under DC-biased magnetization  $(H_{\text{dc}}=107\text{A/m})$  where  $U_{\text{in}}$  and  $J$  are the harmonic vectors of external input voltage and current density, *C* and *Z* represent the coupling matrix and impedance matrix respectively.

> The magnetic field and exciting current can be calculated by solving (9) and (10) together. Fig. 4 shows the calculated exciting current. The fixed-point reluctivity and flux density in one point of the laminated core is presented in Fig. 5.







Fig. 5. Waveform of calculated *ν*<sub>FP</sub> and flux density in one point in the laminated core  $(H<sub>dc</sub>=107A/m, U<sub>ac</sub>=260V)$ 

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